Calibration Report

LEAP-Asia Simulation Exercise – Phase I: Model Calibration

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Model Description

The constitutive model used in this calibration report is the unified plasticity model for large post-liquefaction shear deformation of sand developed by Wang et al. (2014). A brief description of the model is presented here, readers should refer to Wang et al. (2014) for the full formulation of the original model.

The basic equations for the multiaxial model are:

$$\dot{\varepsilon}_v{}^e = \frac{\dot{p}}{K}; \qquad \dot{\mathbf{e}}^e = \frac{\dot{\mathbf{s}}}{2G} \tag{1}$$

$$\dot{\varepsilon}_v^p = \langle L \rangle D; \qquad \dot{\mathbf{e}}^p = \langle L \rangle \mathbf{m} \tag{2}$$

$p = \mathrm{tr}(\boldsymbol{\sigma})/3$ is the mean effective stress, with $\boldsymbol{\sigma}$ being the effective stress tensor; $\mathbf{s} = \boldsymbol{\sigma} - p\mathbf{I}$ is the deviatoric stress, $\mathbf{I}$ being the rank two identity tensor; $\varepsilon_v = \mathrm{tr}(\boldsymbol{\varepsilon})$ is the volumetric strain, $\boldsymbol{\varepsilon}$ being the strain tensor; $\mathbf{e} = \boldsymbol{\varepsilon} - \varepsilon_v/3\mathbf{I}$ is the deviatoric strain tensor. L is the plastic loading index and $\mathbf{m}$ the deviatoric strain flow direction. The deviatoric stress ratio tensor is here defined as $\mathbf{r} = \frac{\mathbf{s}}{p}$, and $q = \sqrt{\frac{3}{2}\mathbf{s}:\mathbf{s}}$, $\eta = \frac{q}{p}$.

The total stress-strain relation can be formulated by combining Eqs. (1) and (2) to be:

$$\dot{\boldsymbol{\varepsilon}} = \frac{1}{2G} p\dot{\mathbf{r}} + \left(\frac{1}{2G}\mathbf{r} + \frac{1}{3K}\mathbf{I}\right)\dot{p} + \left(\mathbf{m} + \frac{D}{3}\mathbf{I}\right)\langle L \rangle \tag{3}$$

with the elastic moduli G and K defined as suggested by Richart et al. (1970):

$$G = G_o \frac{(2.973 - e_{in})^2}{1 + e_{in}} p_a \left(\frac{p}{p_a}\right)^{\frac{1}{2}} \tag{4}$$

$$K = \frac{1 + e_{in}}{\kappa} p_a \left(\frac{p}{p_a}\right)^{\frac{1}{2}} \tag{5}$$

The critical, maximum stress ratio and reversible dilatancy surfaces are shown schematically in Fig. 1.

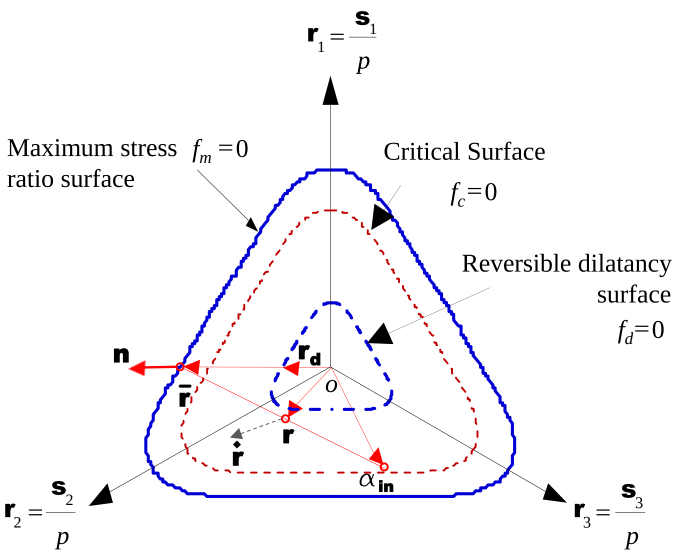

Fig. 1. Schematic illustration of critical state, maximum stress ratio and reversible dilatancy surfaces with mapping rules.

The function $g(\theta)$ in this model is modified based on Zhang's (1997) original proposition:

$$g(\theta) = \left(\frac{1}{1 + M_p(1 + \sin 3\theta - \cos^2 3\theta)/6 + (M_p - M_{p,o})\cos^2 3\theta / M_{p,o}}\right) \quad (6)$$

$$M_p = \frac{6\sin\phi_f}{3 - \sin\phi_f} \quad (7)$$

$$M_{p,o} = \frac{2\sqrt{3}\tan\phi_f}{\sqrt{3 + 4\tan^2\phi_f}} \quad (8)$$

$M_p = M\exp(-n^b\Psi)$ is the peak mobilized stress ratio at triaxial compression and ϕ_f is the corresponding friction angle, $M_{p,o}$ is the peak mobilized stress ratio under torsional shear after isotropic consolidation. The state parameter Ψ proposed by Been and Jefferies (1985) is introduced to consider the dependency of sand behaviour on the current state.

Plastic loading is determined in three dimensional space by the load index L:

$$L = \frac{\mathbf{L} : \dot{\boldsymbol{\sigma}}}{H} = \frac{p\dot{\mathbf{r}} : \mathbf{n}}{H} \tag{9}$$

Here $\mathbf{n}$ is a unit deviatoric tensor serving as the loading direction in deviatoric stress space in the model, and the loading direction $\mathbf{L}$ is defined as $\mathbf{L} = \mathbf{n} - \frac{1}{3}(\mathbf{n} : \mathbf{r})\mathbf{I}$. Plastic loading is induced when $L > 0$, and load reversal occurs at $L < 0$.

It is further assumed that the deviatoric strain flow direction $\mathbf{m}$ in Eq. (3) is the same as the loading direction in deviatoric stress space so as:

$$\mathbf{m} = \mathbf{n} = \bar{\mathbf{r}} / \sqrt{\bar{\mathbf{r}} : \bar{\mathbf{r}}} \tag{10}$$

Here $\bar{\mathbf{r}}$ represents the projection of the current stress point on the maximum stress ratio surface in deviatoric stress space (Fig. 1). The projection of current stress ratio on the maximum stress ratio surface $\bar{\mathbf{r}}$ is defined as the intersection between the extension of the line from the previous load reversal point $\boldsymbol{\alpha}_{\mathbf{in}}$ to $\mathbf{r}$ and the maximum stress ratio surface:

$$\bar{\mathbf{r}} = \boldsymbol{\alpha}_{\mathbf{in}} + \beta(\mathbf{r} - \boldsymbol{\alpha}_{\mathbf{in}}) \tag{11}$$

When the loading index L is positive, plastic loading occurs. Once L becomes negative, load reversal takes place and the projection centre $\boldsymbol{\alpha}_{\mathbf{in}}$ is updated to be the current stress ratio.

The plastic modulus H can then be defined as:

$$H = \frac{2}{3} hG \exp(-n^p \Psi)\left(\frac{M \exp(-n^b \Psi)}{M_m}\left(\frac{\bar{\rho}}{\rho}\right) - 1\right) \tag{12}$$

where $\bar{\rho}$ is the distance between $\bar{\mathbf{r}}$ and $\boldsymbol{\alpha}_{\mathbf{in}}$, and ρ the distance between $\mathbf{r}$ and $\boldsymbol{\alpha}_{\mathbf{in}}$.

The mapping rule for reversible dilatancy is defined so that the projection of the current stress ratio on the reversible dilatancy surface $\mathbf{r_d}$ is the intersection between $\bar{\mathbf{r}}$ and the reversible dilatancy surface:

$$\mathbf{r_d} = \frac{M_d}{M_m}\bar{\mathbf{r}} = \frac{M\exp(n^d\Psi)}{M_m}\bar{\mathbf{r}} \tag{13}$$

According to the propositions made by Shamoto et al. (1997) and Zhang (1997), the dilatancy of sand is decomposed into a reversible and an irreversible component, through which the dilatancy during load reversal and cyclic loading can be properly reflected. In this model, the dilatancy rate D is determined by combining the reversible part D_{re} and irreversible part D_{ir}: The generation and release of reversible dilatancy can then be judged by the angle between $\mathbf{r_d} - \mathbf{r}$ and $\mathbf{n}$:

$$D_{re} = \frac{\dot{\varepsilon}_{vd,re}}{\dot{\gamma}^p} = \begin{cases} D_{re,gen}, & (\mathbf{r_d} - \mathbf{r}) : \mathbf{n} < 0 \\ D_{re,rel}, & (\mathbf{r_d} - \mathbf{r}) : \mathbf{n} > 0 \end{cases} \tag{14}$$

The generation rate of reversible dilatancy is:

$$D_{re,gen} = \sqrt{\frac{2}{3}}d_{re,1}\left(\mathbf{r_d} - \mathbf{r}\right) : \mathbf{n} \tag{15}$$

Reversible dilatancy remains non-positive and is released after load reversal, the release rate is defined as:

$$D_{re,rel} = \left(d_{re,2}\chi\right)^2 / p \tag{16}$$

$d_{re,2}$ is another dilatancy parameter used to calculate the release of reversible dilatancy. $\chi = \min(-d_{ir}\frac{\varepsilon_{vd,re}}{\varepsilon_{vd,ir}^{pr}},1)$ is a function controlling the reversible dilatancy release process, where d_{ir} is an irreversible dilatancy constant and $\varepsilon_{vd,ir}^{pr}$ is the $\varepsilon_{vd,ir}$ at previous load reversal.

Irreversible dilatancy rate D_{ir} defined as:

$$D_{ir}=\frac{\dot{\varepsilon}_{vd,ir}}{\left|\dot{\varepsilon}_q^p\right|}$$
$$=d_{ir}\exp(n^d\Psi-\alpha\varepsilon_{vd,ir})(<M_d-\eta>\exp(\chi)+\left(\frac{\gamma_{d,r}<1-\exp(n^d\Psi)>}{\gamma_{d,r}<1-\exp(n^d\Psi)>+\gamma_{mono}}\right)^2) \tag{17}$$

Here α is a parameter controlling the decrease rate of irreversible dilatancy, γ_{mono} is the shear strain since the last stress reversal and $\gamma_{d,r}$ is a reference shear strain. $\langle\ \rangle$ are the MacCauley backets that yield $\langle x\rangle=x$ if $x>0$ and $\langle x\rangle=0$ if $x\le 0$. The $\exp(n^d\Psi-\alpha\varepsilon_{vd,ir})$ part of the equation reflects asymptotic accumulation of irreversible dilatancy, and the part $\left(\frac{\gamma_{d,r}<1-\exp(n^d\Psi)>}{\gamma_{d,r}<1-\exp(n^d\Psi)>+\gamma_{mono}}\right)^2$ reflects the decreasing dilatancy rate during each monotonic loading process.

Model Parameters

The model parameters are listed in Table 1. Note the parameter $\gamma_{d,r}$ is kept at a default value of 0.05.

Table 1. Model parameters for the simulations.

Sand	G_o	κ	h	M	$d_{re,1}$	$d_{re,2}$	d_{ir}	α	$\gamma_{d,r}$	n^p	n^d	λ_c	e_0	ξ
Ottawa F65	210	0.015	1.2	1.17	0.7	30	0.8	10	0.05	2.25	5.95	0.01	0.7	0.7

Calibration Method

The calibration method for some parameters used in the reported model have been documented by previous researchers, including the elastic modulus constants (G_0, κ), plastic modulus parameter (h) and critical state parameters (M, λ_c, e_0, ξ). The 4 critical state parameters based on Vasko (2015) with modifications to better represent the experiment data given.

The state parameter constants n^p and n^d can be determined through $n^p = \ln(M/\eta_p)/\Psi_p$ and $n^d = \ln(M_d/M)/\Psi_d$, where η_p and Ψ_p are η and Ψ at peak stress ratio in a monotonic drained triaxial test, and M_d and Ψ_d are those at reversible dilatancy sign change points.

Drained cyclic torsional or triaxial tests should be used for the determination of n^d here, as M_d can only be acquired once irreversible dilatancy is negligible after a number of loading cycles. The reversible dilatancy parameters $d_{re,1}$ can be determined using the relationship between η and $\frac{d\varepsilon_{vd}}{d\gamma^p}$ from drained cyclic tests as suggested by Zhang and Wang (2012), and $d_{re,2}$ should then be chosen to ensure the release of reversible dilatancy.

For the irreversible dilatancy parameters (d_{ir} and α especially), a trial-and-error process should be adopted to simulate the stress strain behaviour of undrained cyclic torsional/triaxial tests of different initial confining pressure or shear stress amplitude, as was described by Zhang and Wang (2012). The parameter d_{ir} mainly determines how fast liquefaction is reached in undrained cyclic tests, and α controls the decrease rate of irreversible dilatancy.

The elastic modulus, plastic modulus, and dilatancy parameters are fitted based on three tests with various void ratios and CSRs. Simulations of the rest of the tests are then carried out without any further changes to the model parameters.

Simulation Results

The data for all cyclic torsional test simulations are submitted in spreadsheets along with this report.

References

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